SENSORY PROFILING WITH PROBABILISTIC MULTIDIMENSIONAL SCALING

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ABSTRACT

Variability is a fundamental characteristic of sensory profile data. Ignoring the variability may result in biased solutions that cannot be improved by the collection of additional data. Probabilistic multidimensional scaling (PMDS) models provide a means of accounting for the variability inherent in sensory data by using distributions, instead of points, to portray sensory objects. For profile data with high levels of variability, the probabilistic model recovers latent structure parameters very well — traditional deterministic MDS models and principal components analyses (PCA) do not. Advantages of the PMDS models include their parsimony, testability and extensibility. Two particularly attractive PMDS attributes are their ability to relate consumers' expressions of liking to product profiles and their ability to estimate a product's "perceptual share" from liking and profile data. Used as a criterion with what-if modeling, perceptual share estimates enable the evaluation of alternative product development strategies.

INTRODUCTION

In this article we describe a family of probabilistic multidimensional scaling models that can be used for profiling sensory data and for relating consumers'

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expressions of liking to the latent structures estimated by the models. The proposed models capitalize upon two fundamental characteristics of sensory data, namely, the presence of significant and differential variability in the assessors' profiles and the presence of satiety in consumers' expressions of liking.

MDS models (Borg and Groenen 1997; Young and Hamer 1987) are used to spatially represent the latent structure of objects. If the objects are all real objects, such as food products, then the input data commonly consist of one or more similarity matrices and the traditional output consists of a configuration matrix. In sensory studies, the similarity matrices are frequently derived from profile data by using correlations as direct or distances as inverse measures of similarity. The configuration matrix represents the coordinates of the n objects in a space of r dimensions. If the objects consist of real and ideal products, then the input data are often composed of rectangular matrices of liking ratings. The traditional output consists of a configuration matrix that represents the r coordinates of n real objects and m ideal objects. Ideal objects represent one or more consumers and distances between real and ideal objects are inversely related to the utilities of the consumer(s) for the objects. Estimation of real and ideal object coordinates through the multidimensional scaling of liking ratings is often referred to as unfolding analysis (Coombs 1964). A primary feature of unfolding models is their ability to capture satiety.

The models proposed in this article differ from traditional models in that the latent objects are represented not by points but by distributions. Variances may differ from object to object on any dimension and may also differ across dimensions for any single object. This differential variability of the latent objects comports with the differential variability that is so apparent in the profiling data obtained in sensory studies. Variance estimates are frequently beneficial in the identification of the latent dimensions. They also provide the means for hypothesis testing and permit perceptual share estimation and what-if modeling extensions. A perceptual share is an estimate, based solely upon perceptual data, of what a product's share of market would be. What-if modeling may be used to estimate the perceptual shares that would be obtained by following alternative product development strategies.

In Section 1 we briefly describe the proposed probabilistic scaling models. Section 2 looks at some of the properties of the models and anticipates the benefits that are derived from their use. In Section 3, three real sets of sensory data are described and results obtained with the proposed models are presented. Contrasts to the results obtained using traditional MDS and PCA are reported. Section 4 extends the analysis by using a probabilistic unfolding model to combine the profile results with consumer liking rating data. The probabilistic unfolding model permits estimates of first choice probabilities to be made. Finally, Section 5 discusses the results and demonstrates an application of what-if modeling with PMDS models.

Probabilistic Multidimensional Scaling and Unfolding Models

Traditional MDS models are deterministic; they represent objects as points in a multidimensional space. Coordinates of the points provide the latent structure values of the sensory objects. The latent structure reflects a simplified organization of the manifest (directly observed) variables that are present in the sensory profiles (Rivisk 1996). Coordinates are estimated by optimizing the fit of the distances among the pairs of points in the latent space and the similarities of the sensory profiles. For good discussions on the use of MDS in sensory analysis, see Bieber and Smith (1986), MacFie and Thomson (1984) and Popper and Heymann (1996).

Instead of representing objects as points, our probabilistic model represents the objects as multivariate normal distributions. To relate our spatial model parameters to data, it is convenient to refer to *dissimilarity* and *disutility* data instead of to similarity and preferential choice or liking rating data. A large distance between two real objects indicates that they are dissimilar and a large distance between an ideal object and a real object indicates that the real object has a high disutility or low utility. The normality assumption is common in the development of probabilistic unidimensional models of dissimilarity and disutility (Coombs *et al.* 1961). Specifically, for object S_j, $j=1, \ldots, n$, we let $X_j = (x_{j1}, \ldots, x_{jr})$ be the corresponding r dimensional random vector and assume that it has a r-variate normal distribution with mean vector $\mu_j = (\mu_{j1}, \ldots, \mu_{jr})$ and covariance matrix Σ_j. The $r \times r$ covariance matrix Σ_j may have nonzero off-diagonal values. Constraints may, but need not, be imposed upon the equality of the covariance matrices for different objects or the equality of covariance values for different dimensions. By imposing constraints, simpler models with more degrees of freedom and greater generalizability can often be constructed.

Dissimilarity data δ_{ij} are assumed to be related by three-coefficient linear-exponential measurement model functions to the latent space distances d_{ij}

$$\delta_{ij} = a + b d_{ij}^{c}$$

between objects i and j. The distances

$$d_{ij} = \left(\sum_{k=1}^{r} (x_{ik} - x_{jk})^p \right)^{1/p}$$

are embedded in a Euclidean space when $p = 2$ and a city-block space when $p = 1$.

Alternating maximum likelihood methods provide estimates of the location, variance and measurement model coefficients by maximizing the log of the likelihood function, which is the sum of the logs of the probability density functions (PDFs), over all observations. Estimation proceeds by sequentially reestimating the variances holding the location and measurement estimates fixed, reestimating the location coordinates holding the variance and measurement estimates fixed, and then reestimating the measurement model coefficients, holding the variance and location estimates fixed. This alternating estimation procedure continues to repeat itself until there is no meaningful improvement in the likelihood function. Use of this alternating estimation procedure alleviates the bilateral indeterminacy of scaling (Kruskal 1978) and results in solutions that are as good as or better than those obtained with one-stage approaches.

The PDFs follow a quadratic forms in normal variables distribution (MacKay 1989; Mathai and Provost 1992) for a Euclidean space and a function of folded normal distributions for a city-block space (MacKay 2001). The PDFs of the judgments are derived from the multivariate normality assumptions made about the distributions of the individual objects and the distance functions that define the Euclidean and city-block metrics.

When liking ratings are used, the process is very similar to what has just been described. The primary difference is that in this case, the δ_{ij} values represent disutilities instead of dissimilarities. Objects i and j are thus of different types, one represents an ideal object and the other a real object. The d_{ij} are obtained directly from consumers' liking rating scales in which low numbers represent liking and high numbers represent disliking. An application of PMDS to liking ratings of food products has been reported by MacKay (2001).

Probabilistic Scaling Model — Selected Properties

To discuss the properties of probabilistic scaling models and derive some expectations as to how results derived from probabilistic scaling models will compare both with deterministic MDS models as well as with standard profiling methods such as PCA, it is convenient to have an example. Suppose that 100 consumers are assessing nine yogurts from a 3 × 3 experimental design on two attributes, sweetness and sourness. Let the attributes be of equal importance and suppose that extreme values of the two attributes are evaluated more consistently than mid-range values.

Intensity scale values for consumers' sensory profiles are simulated for this example by independently drawing values for each of the nine products from distributions that represent the two attributes. Let's assume that the mean values for each attribute are 4, 5, and 6 and that the first and third levels of each attribute have a variance of 0.5 and the middle level of each attribute has a variance of 2.5. This scenario is illustrated in the Parameters Panel of Fig. 1 where the letters indicate the centroids of the distributions and the lines represent the equal likelihood contours of the first standard deviational ellipses. The four corner products, *A, C, G, I,* with extreme values on both attributes, have the lowest overall levels of variability and product *E*, with moderate values on both attributes, has the highest.

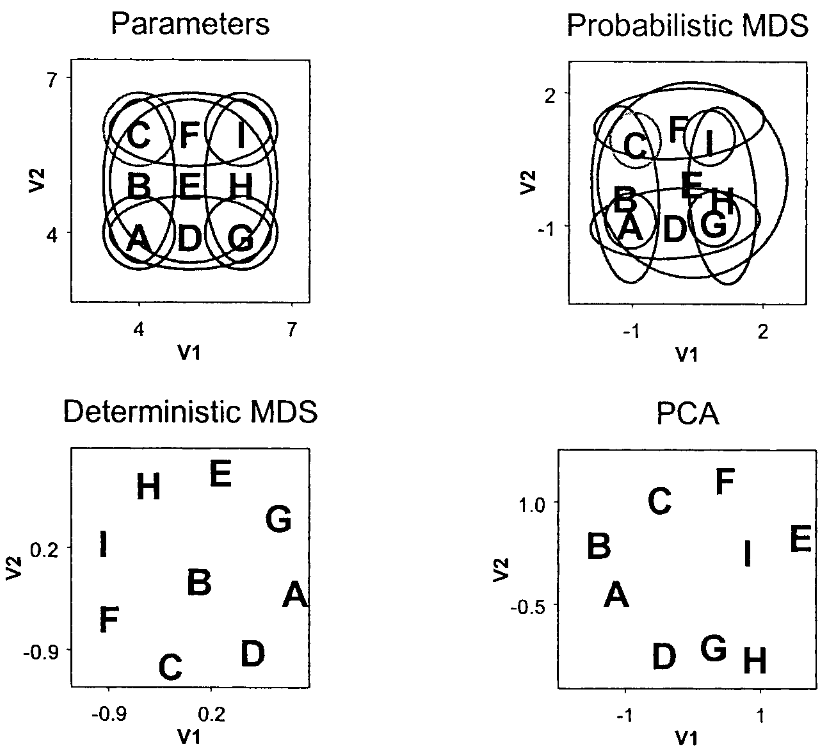

FIG. 1. PARAMETERS AND ESTIMATES FOR THE PROFILE OF NINE PRODUCTS ON TWO ATTRIBUTES

For traditional or probabilistic MDS, dissimilarities are derived using the Euclidean metric from the profiles, one lower-half dissimilarity matrix being computed for each assessor. (Others dissimilarity values have also been used. Bieber and Smith (1986), for example, used Pearson correlation coefficients among profiles as inverse measures of dissimilarity. Cronbach and Gleser (1953) argue for distance measures on the basis of their higher information content).

For PCA, the profile matrices for the 100 assessments may be summed, averaged, or extended by adjoining the 100 9 × 2 profiles horizontally into a 9 × 200 matrix or vertically into a 900 × 2 matrix. When different assessors make assessments or when there is inconsistency in the assessment process, summation or averaging is not appropriate (Gorsuch 1983). Selection of a procedure to adjoin the profiles depends upon the focus of the study. In sensory analyses, where each assessor may have a different number of attributes and where the focus is on obtaining latent space representations for the real products, a horizontal adjoining (9 × 200 in our example) is more common (Bro 1998; Dijksterhuis 1996).

As the products' variances increase, the expected values of the distances between pairs of objects will increase (Patnaik 1949) and the correlations among the manifest variables will change. As correlations and distances depart from the values that would hold in the absence of variance, deterministic models add dimensions in their attempt to accommodate the data. We can thus expect that overestimation of the dimensionality will occur with both MDS and PCA. Probabilistic models, which are able to account for the effects of differential variability, should not have this overestimation problem.

If a correct guess, perhaps from knowledge of the experimental design, as to the dimensionality is made, we can expect that deterministic models will produce biased parameter estimates. In the case of deterministic MDS, the unaccounted for variability will increase the expected distances among the objects, the estimated locations will be spread out and any natural clustering of the products will be diminished. With PCA, the discrepancy of the products' principal component scores and the true latent parameter values should increase. Products with high variances will be estimated as outliers.

An attractive property of probabilistic models is that an error theory is available for testing hypotheses. For deterministic methods, rules of thumb and visual inspection procedures, such as those embodied in scree diagrams, are all that is available. Hypothesis tests can be formed with PMDS models to test for the correct dimensionality of the space, the equality of perceptions, the selection of a metric, and a number of other phenomena as well.

To evaluate the data in our hypothetical example, the PMDS analysis was carried out with PROSCAL (MacKay and Zinnes 2001), a program that permits the scaling of dissimilarity and disutility data under a wide range of conditions. Deterministic MDS analysis was conducted with KYST2 (Kruskal *et al.* 1977), a widely used nonmetric program. PCA solutions were obtained with software from SPSS.

The probabilistic model uses information criterion statistics to test hypotheses. A primary advantage of information criterion statistics over the more traditional likelihood ratio tests is that they allow nonnested models to be compared. This is relevant when testing, for example, city-block and Euclidean metrics that are based upon different PDFs. Information criterion statistics penalize the likelihood by the number of parameters estimated in the model and take the form $-2lnL + cK$ where L is the likelihood, K is equal to the number of free parameters and c is the cost of adding a parameter to the model. When isotropic models (models that assign a common variance to all dimensions of an object) are estimated in a Euclidean space, the number of free parameters K is

$$K=(m+n)r+q-r-\frac{r(r-1)}{2}-1$$

where,

r = the dimensionality of the space
m = the number of real objects
n = the number of ideal objects (0 in the present case)
q = the number of unique variances and covariances being estimated.

The last three terms are subtracted for the centering, rotation, and scale invariance of the solution. When anisotropic models (models that assign unique variances to different dimensions) are used, as they are here, the rotational invariance term is omitted. The rotational invariance term should also be omitted when a city-block metric is used. The cost c of adding a parameter differs among the different information criterion statistics. With Bozdogan's (1987) CAIC criterion, the only widely used criterion that accounts for the effect of sample size, $c = ln(S) + 1$ where S is the sample size. The state of nature with the lowest criterion value is selected.

To begin our probabilistic analysis of the simulated data, solutions were calculated in one, two and three-dimensional spaces. CAIC values are given in Table 1. As anticipated, the correct dimensionality, two, is identified. When the dimensionality is increased beyond three there is little change in the likelihood because the fit is not significantly improved and the values of the CAIC statistic increase because the number of parameters increases, indicating a loss in generalizability.

Results obtained with deterministic MDS and PCA are quite different, as is indicated by the scree diagrams of Fig. 2. The MDS scree plot is very smooth with no apparent elbow. The stress (badness of fit) steadily decreases as the dimensionality increases. The PCA scree plot suggests that a dimensionality of seven or eight would be most appropriate. In either case, there is no evidence that two is the correct dimensionality.

TABLE 1.
DIMENSIONALITY EVALUATION FOR SIMULATED DATA USING A PROBABILISTIC MDS MODEL

Dimensionality	CAIC
1	4823.2
2	3571.3
3	3641.3

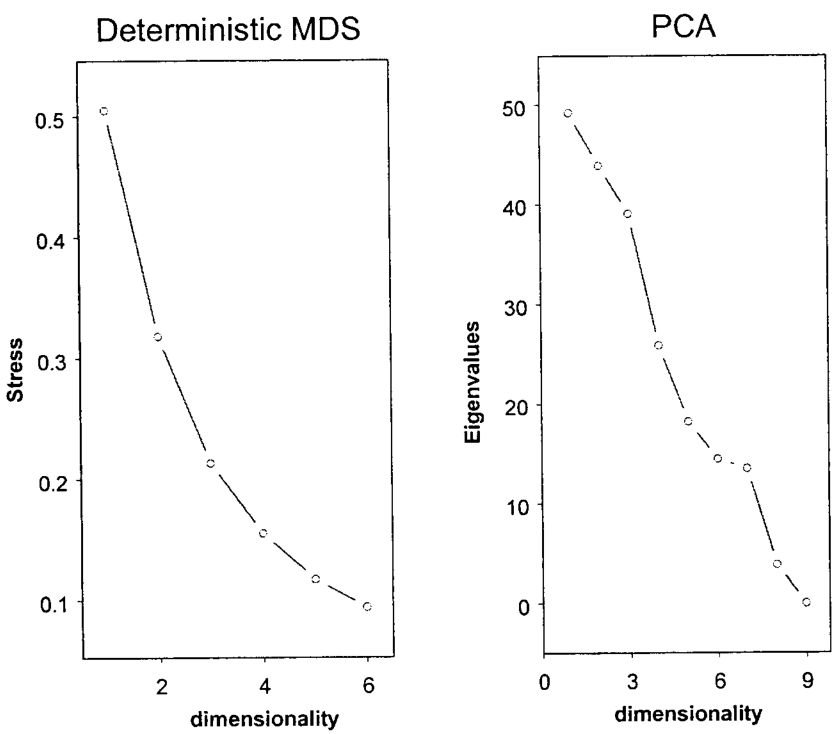

FIG. 2. SCREE DIAGRAMS FOR ASSESSING DIMENSIONALITY OF SIMULATED DATA WITH DETERMINISTIC MDS AND PCA

If two dimensional solutions were investigated, the results would exhibit significant bias for deterministic MDS and PCA. As shown in Fig. 1, the deterministic MDS solution has the products evenly spaced around the perimeter of a circle in an apparently random order with one product in the center of the space. This solution arises because the unaccounted for variance adds to the magnitudes of the expected distances and the solution procedure tries to accommodate this effect by spreading out the points.

In the unrotated PCA solution of Fig. 1, the high variance product, *E*, which should be in the center of the space, is located as an outlier at its periphery. (Varimax and quartimax rotated solutions, not shown, also exhibited the same type of bias. Analyses based on covariance matrices were worse than the reported result which is based upon the correlation matrix.) This biased solution arises because the high variance on product *E* changes the correlational structure of the manifest variables. In this example, if the variances of the products were all small, there would be a distinct pattern to the correlations, odd column variables would have a high correlation with each other, even column variables would have a high correlation with each other, and the correlation of odd and even column variables would be near zero. Increasing product variances has two effects on the correlational structure of the manifest variables. First, the magnitudes of the correlations of odd column variables and the correlations of even column variables diminish. Second, the higher frequency of outliers in the original observations increases the positive and negative magnitudes of correlations among the odd and even column variables. (See Rummel (1970) for a good discussion of the role of manifest variable outliers on observed correlations). In additional simulation studies (not shown), the degree to which *E* is an outlier steadily increased as the uncertainty of the assessors about *E* increased.

In contrast to the deterministic solutions, the PMDS solution (Fig. 1) fares quite well. The general lattice shape of the coordinates is recovered. The estimated standard deviational ellipses, shown in the PMDS panel, comport well with the parameters. The correlation of the standard deviation estimates with the parameters is 0.99 and the correlation of the distances estimated between all 36 pairs of centroid estimates with their corresponding parameters is 0.89. For deterministic MDS and PCA, the correlations for distance estimates and distance parameters are 0.12 and 0.31, respectively.

Differences in standard deviation magnitudes can be due to many things. When assessments are provided by machines or by experts, the standard deviations often reflect quality control differences in the products. When consumers provide assessments, the standard deviations more often reflect differences in perception and difficulties in assigning magnitudes to sensory phenomena. Whatever the cause, the magnitudes of the standard deviations in empirical data are often substantial and ignoring them may cause numerous difficulties.

Data Analyses and Results

Three data sets are explored to see if the expectations described in the prior section on PMDS model properties are fulfilled with empirical data. The first set of data is of eight judges evaluating ten breads on eleven different attributes (Bro 1998). The second data set, from a disguised commercial source, contains 24 expert assessments of 21 snack food products on seven intensity scales. The third set, also from a disguised commercial source, is of 100 consumers evaluating 13 fruit flavored beverages on nine intensity scales. The beverage consumers also provided overall hedonic liking rating judgments that will be looked at in conjunction with the sensory profile results later in this paper.

Bread Data. The ten breads were composed of two replicates of five breads. Four of the breads, pairs *CD, EF, GH* and *IJ* were traditional wheat breads and one, pair *AB*, was a sweet bun. From our knowledge of the products, our expectations are that one dimension should clearly distinguish the sweet buns from the wheat breads and that the replicated pairs of breads should be closer to one another than are other pairs of breads. From the previous section, it is also expected that the PMDS solution should be the most parsimonious.

For the MDS models, dissimilarities were calculated for all 45 pairs of breads for each judge from the attribute data using a Euclidean distance formula. For the PCA analysis, the attribute data were adjoined into a 10×88 matrix. No attribute standardization was performed since all of the attributes were in the same units and range.

Table 2 reports the CAIC statistics from the dimensionality test of the PMDS model. A two dimensional solution is indicated. Figure 3 reports the scree tests of the deterministic MDS model and the PCA model. As with the simulated data, the deterministic MDS model gives little clue as to the appropriate dimensionality. The stress value steadily diminishes as additional dimensions are estimated. For the PCA analysis, the scree plot suggests a dimensionality of two while the Kaiser criterion (the number of eigenvalues greater than 1.0) suggests a dimensionality of nine.

The initial latent profile for the probabilistic scaling model was estimated with 20 unique variances, two for each of the ten breads. A visual inspection suggested that the variances for the two sweet buns were approximately the same and the variances for the eight wheat breads were approximately the same. Accordingly, a constrained solution with four unique variances, two for the sweet buns and two for the wheat breads, was estimated. The CAIC statistic was used to see if the diminished log likelihood for the constrained solution was outweighed by the gain in generalizability. The CAIC statistic was lower for the constrained solution so it was used instead of the original unconstrained solution.

TABLE 2.
DIMENSIONALITY EVALUATION FOR BREAD DATA USING A PROBABILISTIC MDS MODEL

Dimensionality	CAIC
1	1407.7
2	1403.5
3	1474.1

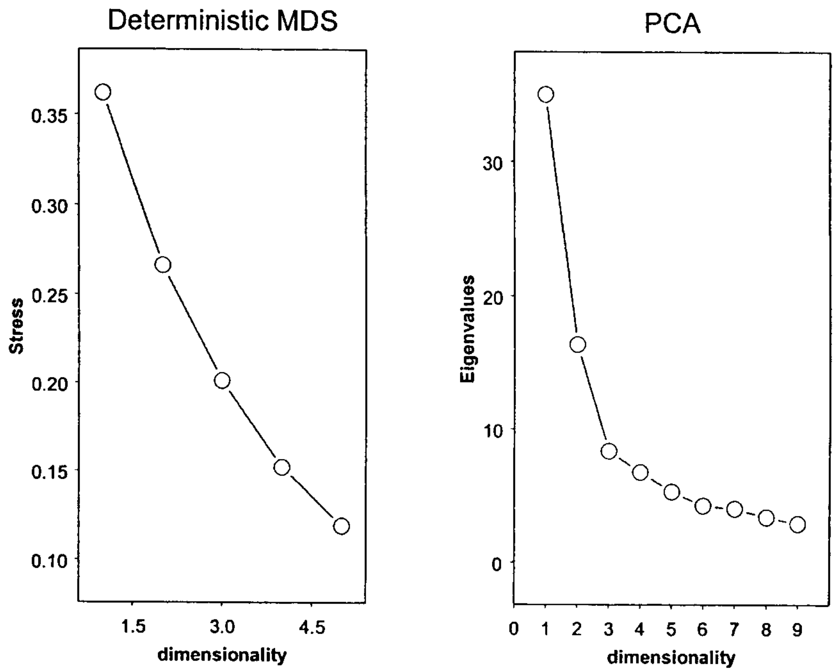

FIG. 3. SCREE DIAGRAMS FOR ASSESSING DIMENSIONALITY OF BREAD DATA WITH DETERMINISTIC MDS AND PCA

Figure 4 presents two-dimensional solutions for the probabilistic and deterministic MDS analyses and the solutions for two PCA analyses. It is interesting to note that the variance structure for the sweet buns and wheat breads suggests that the judges are very sensitive to values on the first axis, presumably sweetness, for the sweet buns but relatively insensitive to sweetness when it comes to the whole wheat breads. Differences among the wheat breads, measured on the second axis, reflect differences in saltiness, toughness and moisture (Bro 1998).

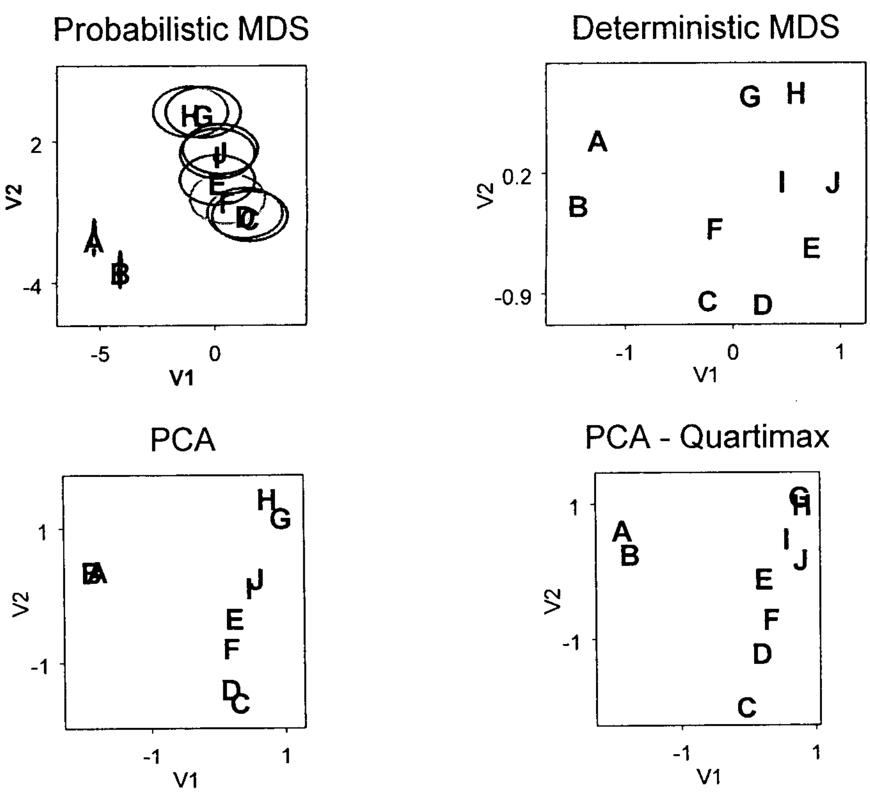

FIG. 4. ESTIMATED LATENT PRODUCT PROFILES FOR BREAD DATA

Differences in the PMDS and MDS solutions correspond with our expectations from the simulation analysis, the deterministic solution is characterized by points which are more spread out than the more highly clustered probabilistic solution. However, there is no indication of an outlier with either of the PCA solutions.

Expected distances were calculated for all pairs of products from the PMDS solution and interpoint distances were calculated for all pairs of products with the other three solutions. For the PMDS solution, the five product pairs with the smallest expected distances were, as anticipated, the five replicated pairs. The same was true for the correlation based PCA solution. This was not true of the distances for the deterministic MDS solutions. The four pairs with the smallest interpoint distances with the deterministic MDS solution were replicated pairs but pair *EF* was fourteenth.

Additional PCA solutions were derived using quartimax and varimax rotations. For the PCA quartimax solution, the top three pairs were replicated pairs but pair *EF* was sixth and pair *CD* was eleventh. For the PCA varimax solution (not shown), three of the top four pairs were replicated pairs but pair *GH* was ninth and pair *CD* was twentieth.

Snack Food Data. The 21 snack food products were evaluated on seven taste and texture attributes. High levels of variability were observed across attributes and across products by attribute.

A comparison of the two-dimensional PMDS and PCA solutions for this set of data provide a very nice illustration of the false outlier effect that can occur when sensory profile data are evaluated with PCA. PMDS and PCA solutions are provided in Fig. 5.

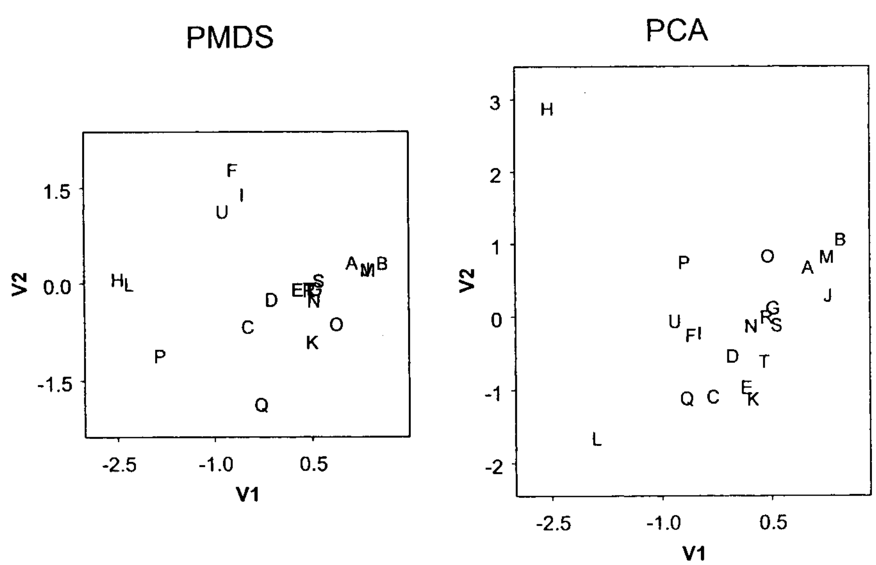

FIG. 5. ESTIMATED LATENT PRODUCT PROFILES FOR SNACK FOOD DATA

The two configurations, while bearing some resemblance to each other, are much more different than the configurations compared with the bread data. Here, a comparison of the intercentroid distances of the PMDS solution and the interpoint distances of the PCA solution yields a correlation of 0.64. A major contributor to this relatively low correlation is the difference in the estimated locations of products *H* and *L* which, in the PMDS solution, are very close to each other and in the PCA solution are at opposite ends of the second dimension.

Not knowing the parameters of this data set, we cannot say for certain that the PCA solution is an example of the false outlier phenomenon but several things point to this being the case. With the PMDS solution, the largest variance estimates (not illustrated to enhance the clarity) on the second dimension are for products *H* and *L*, 6.372 and 3.961, respectively. The next highest variance, for product *C*, is 3.324 and all of the other variances are less than 2.2. Going back to the original manifest variables, a nearest neighbor inspection was conducted by calculating the mean absolute differences of the seven attributes between each product and the other 20 products. For product *L*, *H* was its nearest neighbor and for product *H*, *L* was its second nearest neighbor. Products *H* and *L* thus have variances which may produce outliers while summary measures of the original judgments suggest that *H* and *L* should be close together.

Just as we made use of the error theory that accompanies PMDS to determine the best dimensionality and appropriate variance structure with the bread data, we can use the error theory to test the hypothesis that the centroids of products *H* and *L* are the same. The log likelihood for the PMDS outcome of Fig. 5 is -1668.1. When a constrained solution that required the centroids of products *H* and *L* to be the same was computed the resulting log likelihood was -1668.2. The two values are exceptionally close and the CAIC statistic indicates that the constrained solution is preferred.

Finally, if instead of adjoining the profile data, the profiles are averaged, then the PCA solution, like the PMDS solution, estimates the locations of *H* and *L* as being very close to one another. From this observation might come the question "why then should we adjoin the manifest variable data?" One response to this of course is that while the two products are seen as similar when all judgments are averaged, at the individual assessment level, they are seen as quite different. Only the PMDS solution captures this phenomenon of highly variable judgments and similar means. The PCA and MDS (not shown) solutions confound location estimation and variability and leave the false impression that the two products are fundamentally different. A second response is that by estimating a variance structure with PMDS, we are then able to combine profile data with liking rating data and estimate perceptual shares, as we illustrate with the next data set.

Beverage Data. Unlike the earlier data sets, the sensory profile data for this set of 13 fruit flavored beverages came from consumers. Each of the nine attributes was evaluated on a nine point intensity scale. As before, Euclidean distances were derived from the profiles for the MDS analyses.

We begin with an investigation of dimensionality. The PMDS model, which was estimated with unique variances for each product, identified the correct dimensionality as four using the CAIC statistic. Scree tests (not shown) were indeterminate in the case of deterministic MDS and indicated a 12 dimensional space in the case of PCA. For ease of exposition, we shall describe the two dimensional solution and comment, where necessary, on the differences apparent in the four dimensional solution.

PMDS, deterministic MDS and PCA solutions are given in Fig. 6. The general form of the PMDS and PCA solutions are, as with the bread data set, very similar while the deterministic MDS solution is again degenerate with a configuration that is determined largely by the need to accommodate distances that are determined in large part by the variability in the assessors' judgments. The PMDS panel suggests that a simpler solution, with common variances for products *K* and *M* and common variances for the other products might be feasible. This time, though, the hypothesis of a simpler variance structure was rejected with the CAIC test statistic. The location of the centroids and the variance structure apparent in Fig. 6 also held for the first two dimensions of the four dimensional configuration. The third dimension was generally planar with point *H* above the plane. The fourth dimension was also generally planar with point *B* below the plane.

Dimension identification is aided considerably by knowledge of the dimensional variances. Products *K* and *M*, whose variance structure is different from all of the other products, are the two products with the lowest level of consumer perceived sweetness. Their level of sweetness is uniformly perceived whereas for the other products variability in sweetness perception is very high. Dimension V2 thus appears to be an "unsweetness" dimension. Dimension V1 has negative correlations below -0.9 ($P \leq 0.01$) with the profile variables of cloudiness, aroma, flavor and thickness. This dimension we shall thus refer to as "weakness". (Low values of dimension three indicated a strong color of the beverages and dimension four measured tartness).

Summary. All three data sets were similar to the simulated data in that there was a significant divergence between the PMDS and deterministic MDS solutions. The differences are caused by the deterministic MDS solutions being unable to accommodate the differential variation inherent in the dissimilarity values derived from the assessors' product profiles. Like the simulated data, the snack food data set exhibited a marked outlier effect in its estimation of latent values for two products having a high level of variation. However, outlier effects were not observed with the bread and beverage data sets. If the products in the bread and beverage studies had been more complex or the assessors more uncertain, the degeneracy observed in the PCA with the simulated data and the snack food data would have been observed here as well. Finally, in having as low or lower an estimated dimensionality as deterministic MDS or PCA, the PMDS solutions were shown to be more parsimonious.

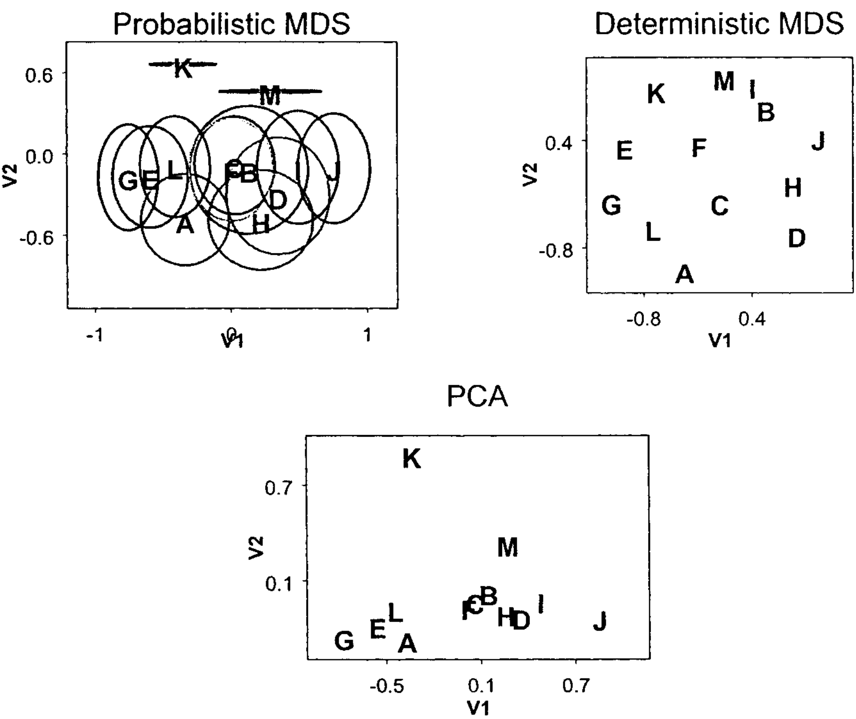

FIG. 6. ESTIMATED LATENT PRODUCT PROFILES FOR BEVERAGE DATA

External Unfolding Analysis

A major advantage of PMDS models is their ability to incorporate the results of sensory profile analysis with consumers' liking ratings of the products. Given the estimated distributions of the real products, as shown in Fig. 6, the task is to unfold the unidimensional disutility judgments (liking ratings) into the externally defined multidimensional product space by estimating distributions for one or more ideal products. The analyst may define the number and membership of ideal distributions by fiat or by using multivariate methods. (See Arabie *et al.* (1996) for an overview of much of the recent statistical and data mining literature on cluster analysis).

In this application, since there was no *a priori* basis for determining the segments, the nine point liking rating scale data were standardized for each subject across the evaluated products and submitted to sequential K-means cluster analyses. Unfolding analyses were conducted for two through six-cluster solutions. A four-ideal product solution was chosen because the clusters were of approximately equal size and the estimated distributions were distinct from each other. Figure 7 shows the estimated distributions for the ideal products. Centroids for the real products are shown as well, but the standard deviational ellipses, given in Fig. 6, are omitted to enhance the clarity. Ideal products 1 and 3 each represent 28% of the subjects; ideal product 2 represents 21% and ideal product 4 represents 23%.

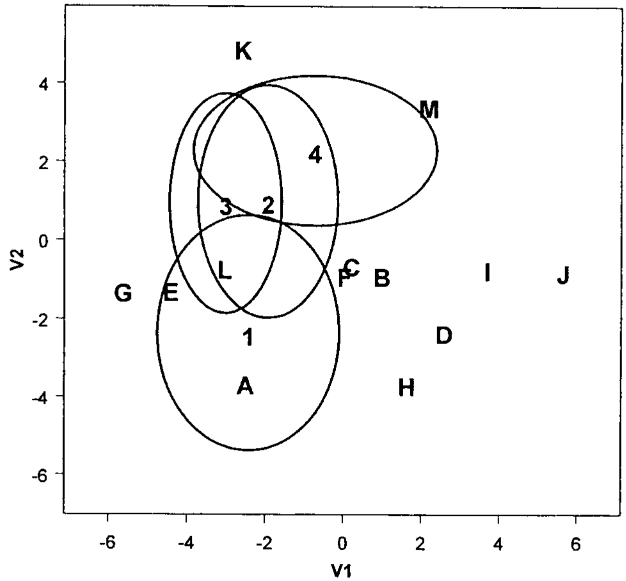

FIG. 7. ESTIMATED DISTRIBUTIONS FOR FOUR MARKET SEGMENTS AND CENTROIDS FOR THIRTEEN REAL PRODUCTS

Several things about Fig. 7 are worth noting. The four segments are distinguished more by differences in their preferences for sweetness than they are in their preferences for different strength/weakness levels. Segment 4, the segment with the strongest preference for unsweetened products, has a higher sensitivity to sweetness, as shown by the smaller standard deviation on that dimension, than do the other segments. Segment 1 prefers sweetened products while Segments 2 and 3 prefer products with moderate levels of sweetness. None of the segments have a preference for "weak" products. This is borne out in the raw liking rating data where the five least liked products are *D, H, I, J* and *M*.

Given the distributions of Fig. 6 and 7, it is possible to define joint density functions over the product space for each segment. Numerical integration or simulation can then be used to calculate the proportion of first choices for each real product by segment. These proportions can be weighted by the sample sizes of the segments to come up with overall probability of first choice estimates for all thirteen products.

Each product's perceptual share, the probability that a product will be chosen first by consumers, can be calculated under the assumption of independent or dependent sampling. When independent sampling is assumed, it is as if in evaluating each real product, a subject samples new values from his or her ideal distribution for each comparison. When dependent sampling is assumed, it is as if only one sample were drawn from a subject's ideal distribution and that those sample values, one for each dimension, were then compared to samples drawn from each of the real product distributions. The probability of choosing one product over another is thus independent of the other alternatives with independent sampling and contingent upon the other alternatives with dependent sampling.

For the beverage data, overall first choices under dependent sampling are highly correlated with first choices under independent sampling ($r = 0.94$, $P \leq 0.01$). When, though, you look at the individual segments, some marked differences occur. Under independent sampling, for example, the expected distances of products *F, C* and *K* to ideal segment 2 are about the same and the estimated first choice shares of products *F, C* and *K* are 0.12, 0.12 and 0.11. Under dependent sampling, though, products *F* and *C* compete for the same share and the first choice probabilities of *F, C* and *K* are quite different -0.10, 0.10 and 0.17.

Selection of a sampling method may be made by fiat or on the basis of fit with the data. In this application, dependent sampling produced first choice probabilities that had a higher overall fit with the first choices of the consumers. As expected, the overall first choice shares under dependent sampling are also highly correlated with the mean liking ratings ($r = 0.83$, $P \leq 0.01$).

Discussion and What-If Modeling Application

In this article we have shown that variability in assessors' sensory profiles of products can result in degenerate solutions for both deterministic MDS and PCA. Deterministic MDS is more sensitive to differential variability than PCA but both methods will, if the variability is high enough, produce meaningless results. In contrast, PMDS, which explicitly models product variability, is more robust in the presence of variation in assessor profiles.

Having a statistical foundation, PMDS has the ability to formally test hypotheses. The solutions estimated with PMDS are generally more parsimonious, of lower dimensionality, than are those estimated either with deterministic MDS or PCA.

In addition to providing an error theory that enables hypothesis testing, the variance structure of the PMDS models is useful in identifying the latent dimensions of the reduced space and in estimating the first choice probabilities of the products being profiled, both for individual market segments and the entire market.

First choice probabilities provide a convenient way to use the output of PMDS models for what-if modeling. Suppose, for example, that the thirteen products of Fig. 6 and 7 are the leading products in their market category. Several developmental strategies are suggested by the output of Fig. 6 and 7. One strategy would be to take the most popular product, L, whose first choice probability of 0.15 dominates the category, and create a copy-cat product. Suppose that this is accomplished and that the new product has the same location and variance parameters as L. This strategy should be quite successful, the new product and product L would end up with perceptual shares of almost 0.11 each.

An alternative strategy would be to take the largest segment, say it is segment 1 (segments 1 and 3 both have 28% of the market) and create a product whose centroid is the same as that for segment 1. Let us suppose that this is accomplished and that the variance structure of the new product is the same as the product nearest to the centroid of segment 1, product A. This strategy would be less successful than the copy-cat strategy, netting a perceptual share of 0.09.

It is often profitable to go further with what-if modeling efforts and ask questions that will help determine a range of possible outcomes. If, for example, the desired product is not successfully introduced and the perceived variances are much larger than anticipated, what will the effect be on the estimated perceptual share?

Finally, it must be stressed that the share estimates being estimated are perceptual shares, not market shares. Perceptual share estimates are based only upon consumers' perceptions of the products. They do not account for the many things, competitive responses, brand equity, marketing variables, changing economic conditions, etc., that mediate consumers' shopping decisions.

Integrating the effects of these factors with the output from sensory analysis is possible but goes beyond the scope of this article.

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